# About Constructions of Reactions of Servo-Constraints of Body.


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***Abstract:*** *The problem of vibro protection a body having six degrees of freedom is considered. Considering, that on system is imposed six servo constraints in view of parametrically clearing system from servo constraints, the equations of motion of a body are received. Conditions, at which stability of system under the relation of the variety, determined by servo constraints is provided, are found.*



***Key words:*** *active vibro protection, the shock-absorber, servo constraint, (A) – moving, clearing parameters, dempfering, equations of motion, stability, the differential equations, stability conditions.*


Let's consider a body in weight *m*, suspended to the basis on shock-absorbers. Let on a body are imposed servo constraints [1]:

$$\left.\begin{aligned} q_1 = 0, \quad q_2 = 0, \quad q_3 = 0, \\ \varphi_1 = 0, \quad \varphi_2 = 0, \quad \varphi_3 = 0 \end{aligned}\right\} \qquad (1)$$

where $q_1, q_2, q_3, \varphi_1., \varphi_2., \varphi_3$-generalized coordinates.

It is known [1-3], that along with (1) parities also take place:

$$\left.\begin{aligned} q_1 = \xi_1, \quad q_2 = \xi_2, \quad q_3 = \xi_3, \\ \varphi_1 = \xi_4, \quad \varphi_2 = \xi_5, \quad \varphi_3 = \xi_6 \end{aligned}\right\} \qquad (2)$$

where $\zeta_{1,} \zeta_2, \ldots, \zeta_6$ - the independent parameters, characterizing continuous clearing of system from servo constraints (1). Movings, on which servo constraints work do not make reaction [1-3], look like:

$$\left.\begin{aligned} \delta\xi_1 = 0, \quad \delta\xi_2 = 0, \quad \delta\xi_3 = 0, \\ \delta\xi_4 = 0, \quad \delta\xi_5 = 0, \quad \delta\xi_6 = 0 \end{aligned}\right\} \qquad (3)$$

If to neglect weight and dempfer of the shock-absorbers, and body displacement to consider small enough, motions, of such one-mass system can be described by six the differential equations of the second order [4]:

1) $\grave{o}\,\ddot{\xi}_1 + \sum_{i=1}^{4} C_{xi}\xi_1 + \sum_{i=1}^{4} C_{xi} z \cdot \xi_5 - \sum_{i=1}^{4} C_{xi} y \cdot \xi_6 = Q_1 \cdot \cos(\Omega t + \psi_1) + \wedge_1$ .

2) $\grave{o}\,\ddot{\xi}_2 + \sum_{i=1}^{4} C_{yi}\xi_2 - \sum_{i=1}^{4} C_{yi} z \cdot \xi_4 + \sum_{i=1}^{4} C_{yi} x \cdot \xi_6 = Q_2 \cdot \cos(\Omega t + \psi_2) + \wedge_2$ .

3) $m\ddot{\xi}_3 + \sum_{i=1}^{4} C_{zi}\xi_3 + \sum_{i=1}^{4} C_{zi} y \cdot \xi_4 - \sum C_{zi} x \cdot \xi_5 = Q_3 \cdot \cos(\Omega t + \psi_3) + \wedge_3$ .

4) $I_x\ddot{\xi}_4 - I_{xy} \cdot \ddot{\xi}_5 - I_{zx}\ddot{\xi}_6 - \sum_{i=1}^{4} C_{yi} \cdot z \cdot \xi_2 + \sum_{i=1}^{4} C_{zi} y \cdot \xi_3 + \sum_{i=1}^{4} (C_{zi} \cdot y^2 - C_{yi} z^2) \cdot$

$\cdot \xi_4 - \sum_{i=1}^{4} C_{zi} xy \cdot \xi_5 - \sum_{i=1}^{4} C_{yi} xy \cdot \xi_6 = M_1 \cdot \cos(\Omega t + \psi_4) + \wedge_4$ .

5) $I_y\ddot{\xi}_5 - I_{xy} \cdot \ddot{\xi}_4 - I_{yz}\ddot{\xi}_6 + \sum_{i=1}^{4} C_{xi} z \cdot \xi_1 - \sum_{i=1}^{4} C_{zi} x \cdot \xi_3 - \sum_{i=1}^{4} C_{zi} xy \cdot$

$\cdot \xi_4 + \sum_{i=1}^{4} (C_{xi} z^2 + C_{zi} x^2) \cdot \xi_5 - \sum_{i=1}^{4} C_{xi} y \cdot \xi_6 = M_2 \cdot \cos(\Omega t + \psi_5) + \wedge_5$ .

$$6)\ I_z \cdot \ddot{\xi}_6 - I_{xy}\ddot{\xi}_4 - I_{yz}\ddot{\xi}_5 - \sum_{i=1}^{4} C_{xi}\, y \cdot \xi_1 + \sum_{i=1}^{4} C_{yi}\, x \cdot \xi_2 - \sum_{i=1}^{4} C_{yi}\, xy \cdot$$
$$\cdot \xi_4 - \sum_{i=1}^{4} C_{xi}\, zy \cdot \xi_5 + \sum_{i=1}^{4} (C_{xi} y^2 + C_{yi} x^2)\xi_6 = M_3 \cdot \cos(\Omega t + \psi_3) + \wedge_6 \qquad (4)$$

In the equations (4), representing forces and the moments, for convenience of using for the generalized co-ordinates, are accepted: $q_1, q_2, q_3$ - linear motions on axes $x, y, z$; $\varphi_1, \varphi_2, \varphi_3$ - for angular turns round the axes which beginning coincides with the block centre of gravity. The main axes of inertia of vibro protectioning the block, are directed on the same axes. Other parameters entering into the equations (4):

$m$ - weight of vibro protectioning the block; $\overline{C}_x, \overline{C}_y, \overline{C}_z$ - operational factors of rigidity;

$I_x, I_y, I_z I_{xy}, I_{xz}, I_{yz}$ - the moments of inertia of vibro protectioning the block; $Q_1, Q_2, Q_3, M_1, M_2, M_3$ - components of external forces and the moments, operating on vibro protectioning the block on corresponding axes; $\wedge_1, \wedge_2, \ldots, \wedge_6$ - reactions of servo constraints.

From expressions (4) it is visible that at block installation on four shock-absorbers the system possesses six degrees of freedom. Here as the generalized system the co-ordinates, characterizing motions of object concerning the basis are chosen. It is convenient, as for active vibro protection relative motions coincide with absolute, and for passive vibro protection, considering known position of dynamics of relative motion, it is possible to consider basis motion, entering forces of inertia in portable motion. Forces of Koriolis by consideration of small fluctuations can be rejected, as they are proportional to products of small portable angular speeds for small speeds in relative motion of system, are equally suitable both for active, and for passive vibro protection systems. Using a constructive method of search of structure of forces of reactions of servo constraints (Method of A.G.Azizov) [1-3], if reaction of servo constraints to form under laws,

$$\wedge_1 = \sum_{i=1}^{4} C_{xi}\xi_1 + \sum_{i=1}^{4} C_{xi} z \cdot \xi_5 - \sum_{i=1}^{4} C_{xi} y \cdot \xi_6 - Q_1 \cdot \cos(\Omega t + \psi_1) - k_{11}\dot{\xi}_1 - k_{12}\xi_1,$$
$$\wedge_2 = \sum_{i=1}^{4} C_{yi}\xi_2 - \sum_{i=1}^{4} C_{yi} z \cdot \xi_4 + \sum_{i=1}^{4} C_{yi} x \cdot \xi_6 - Q_2 \cdot \cos(\Omega t + \psi_2) - k_{21}\dot{\xi}_1 - k_{22}\xi_2,$$
$$\wedge_3 = \sum_{i=1}^{4} C_{zi}\xi_3 + \sum_{i=1}^{4} C_{zi} y \cdot \xi_5 - \sum_{i=1}^{4} C_{zi} x \cdot \xi_5 - Q_3 \cdot \cos(\Omega t + \psi_3) - k_{31}\dot{\xi}_3 - k_{32}\xi_3,$$
$$\wedge_4 = -\sum_{i=1}^{4} C_{yi} z \cdot \xi_2 + \sum_{i=1}^{4} C_{zi} y \cdot \xi_3 + \sum_{i=1}^{4} (C_{zi} y^2 - C_{yi} x^2)\xi_4 - \sum_{i=1}^{4} C_{zi} xy \cdot \xi_5 -$$
$$\sum_{i=1}^{4} C_{yi} xy \cdot \xi_6 - M_i \cos(\Omega t + \psi_4) - k_{41}\dot{\xi}_4 - k_{42}\xi_4 \quad ,$$
$$\wedge_5 = \sum_{i=1}^{4} C_{xi} z \cdot \xi_1 - \sum_{i=1}^{4} C_{zi} x \cdot \xi_3 - \sum_{i=1}^{4} C_{zi} xy \cdot \xi_4 + \sum_{i=1}^{4} (C_{xi} z^2 + C_{zi} x^2) \cdot \xi_5 -$$
$$\sum_{i=1}^{4} C_{xi} y \cdot \xi_6 - M_2 \cos(\Omega t + \psi_5) - k_{51}\dot{\xi}_5 - k_{52}\xi_5 \quad ,$$
$$\wedge_6 = \sum_{i=1}^{4} C_{xi} y \cdot \xi_1 + \sum_{i=1}^{4} C_{yi} x \cdot \xi_2 - \sum_{i=1}^{4} C_{yi} xy \cdot \xi_4 -$$
$$-\sum_{i=1}^{4} C_{xi} zy \cdot \xi_5 + \sum_{i=1}^{4} (C_{xi} y^2 + C_{yi} x^2) \cdot \xi_6 - M_3 \cos(\Omega t + \psi_6) - k_{61}\dot{\xi}_6 - k_{62}\xi_6 \quad , \qquad (5)$$

where $k_{11}, k_{12}, \ldots, k_{61}, k_{62}$ - some constants, substituting (5) in (4), we will receive the equations of the indignant motions:

$$\begin{cases} m\ddot{\xi}_1 + k_{11}\dot{\xi}_1 + k_{12}\xi_1 = 0 \\ m\ddot{\xi}_2 + k_{21}\dot{\xi}_2 + k_{22}\xi_2 = 0 \\ m\ddot{\xi}_3 + k_{31}\dot{\xi}_3 + k_{32}\xi_3 = 0 \\ I_x \cdot \ddot{\xi}_4 - I_{xy} \cdot \ddot{\xi}_5 - I_{zx}\ddot{\xi}_6 + k_{41}\dot{\xi}_4 + k_{42}\xi_4 = 0 \\ I_y \cdot \ddot{\xi}_5 - I_{xy} \cdot \ddot{\xi}_4 - I_{yz}\ddot{\xi}_6 + k_{51}\dot{\xi}_5 + k_{52}\xi_5 = 0 \\ I_z \cdot \ddot{\xi}_6 - I_{xy}\ddot{\xi}_4 - I_{yz}\ddot{\xi}_5 + k_{61}\dot{\xi}_6 + k_{62}\xi_6 = 0 \end{cases} \qquad (7)$$

System (7) has the private decision,

$$\xi_1 = 0, \quad \xi_2 = 0, \quad \xi_3 = 0,$$
$$\xi_4 = 0, \quad \xi_5 = 0, \quad \xi_6 = 0, \qquad (8)$$

corresponding the parities (1).

As it is known, for stability of the decision (8) it is necessary and enough that all roots of the characteristic equation had negative material parts [5]. As factors of the equation constant numbers last conditions can be received using Hurvits's criterion [5]. We Investigate at what values of constants $k_{11}, k_{21}, \ldots, k_{61}$ stability of the zero decision (8) systems (7) is provided. For this purpose we will make a characteristic determinant of system (7):

$$\begin{vmatrix} \grave{o}\lambda^2 + \hat{e}_{11}\lambda + \hat{e}_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & \grave{o}\lambda^2 + \hat{e}_{21}\lambda + \hat{e}_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \grave{o}\lambda^2 + \hat{e}_{31}\lambda + \hat{e}_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x\lambda^2 + \hat{e}_{41}\lambda + \hat{e}_{42} & -I_{xy}\cdot\lambda^2 & -I_{zx}\lambda^2 \\ 0 & 0 & 0 & -I_{xy}\cdot\lambda^2 & I_y\lambda^2 + \hat{e}_{51}\lambda + \hat{e}_{52} & -I_{yz}\lambda^2 \\ 0 & 0 & 0 & -I_{xy}\lambda^2 & -I_{yz}\lambda^2 & I_z\lambda^2 + \hat{e}_{61}\lambda + \hat{e}_{62} \end{vmatrix}$$

The characteristic equation of system looks like:

$$(m\lambda^2 + \kappa_{11}\lambda + \kappa_{12})(m\lambda^2 + \kappa_{21}\lambda + \kappa_{22})(m\lambda^2 + \kappa_{31}\lambda + \kappa_{32})$$
$$(I_x\lambda^2 + \kappa_{41}\lambda + \kappa_{42})(I_y\lambda^2 + \kappa_{51}\lambda + \kappa_{52})(I_z\lambda^2 + \kappa_{61}\lambda + \kappa_{62}) = 0 \qquad (9)$$

which it is led to a kind:

$$m\lambda^2 + \kappa_{11}\lambda + \kappa_{12} = 0$$
$$m\lambda^2 + \kappa_{21}\lambda + \kappa_{22} = 0$$
$$m\lambda^2 + \kappa_{31}\lambda + \kappa_{32} = 0$$
$$I_x\lambda^2 + \kappa_{41}\lambda + \kappa_{42} = 0$$
$$I_y\lambda^2 + \kappa_{51}\lambda + \kappa_{52} = 0$$
$$I_z\lambda^2 + \kappa_{61}\lambda + \kappa_{62} = 0 \qquad (9a)$$

Roots of the characteristic equation (9) or (9a)

$$\lambda_i = \frac{-k_{i1} \pm (k_{i1}^2 - 4 \cdot m \cdot k_{i2})^{1/2}}{2 \cdot m},$$

$$\lambda_{3+i} = \frac{-k_{3+i,1} \pm (k_{3+i,1}^2 - 4 \cdot I \cdot k_{3+i,2})^{1/2}}{2 \cdot I}, \quad (i = 1; 2; 3)$$

we will choose such, that they had negative material parts, i.e. performance of conditions is required:

$$k_{i1} > 0, \quad k_{3+i,1} > 0, \quad i = 1,2,3 \qquad (10)$$

In real of shock -absorber systems on degree of freedom of the block certain constraints or the conditions [6], simplifying system of the equations are imposed. For example, at rest the block has no warps and the centre of rigidity of shock-absorbers (CR) lies on one vertical with the centre of gravity (CG) and in system (4) third equation becomes independent. Besides, rigidity on axes and distance from the centre of gravity on an axis to deformable elements are usually identical [7]. As axes are the main central axes of inertia, the centrifugal moments are equal to zero and the system of the known equations (4) breaks up to following four groups:

$$I.II\begin{cases} m\xi_3'' + \sum_{i=1}^{4} \overline{C}_{zi}\xi_3 = Q_3\cos(\Omega t + \Psi_3) + \Lambda_3 \\ I_z\xi_6'' + \sum_{i=1}^{4} \left(C_{xi}y^2 + C_y x^2\right)\xi_6 = M_3\cos(\Omega t + \Psi_6) + \Lambda_6 \end{cases} \quad (11)$$

$$\text{III.}\begin{cases} m\xi_1'' + \sum_{i=1}^{4} \overline{C}_{xi}\xi_1 + \sum \overline{C}_{xi} z\xi_5 = Q_1\cos(\Omega t + \Psi_1) + \Lambda_1 \\ I_y\xi_5'' + \sum_{i=1}^{4} \overline{C}_{xi} z\xi_1 + \sum\left(C_{xi}z^2 + C_z x^2\right)\xi_5 = M_2\cos(\Omega t + \Psi_5) + \Lambda_5 \end{cases} \quad (12)$$

$$IV\begin{cases} m\xi_2'' + \sum_{i=1}^{4} \overline{C}_{yi}\xi_2 - \sum \overline{C}_{yi} z\xi_4 = Q_2\cos(\Omega t + \Psi_2) + \Lambda_2 \\ I_x\xi_4'' - \sum_{i=1}^{4} \overline{C}_{yi} z\xi_2 + \sum_{i=1}^{4}\left(\overline{C}_{zi}y^2 + \overline{C}_y z^2\right)\xi_4 = M_1\cos(\Omega t + \Psi_4) + \Lambda_4 \end{cases} \quad (13)$$

If forces of reactions of servo constraints to form under the law:

$$\wedge_1 = \sum_{i=1}^{4} C_{xi}\xi_1 + \sum_{i=1}^{4} C_{xi} z\cdot\xi_5 - Q_1\cdot\cos(\Omega t + \psi_1) - k_{11}\dot{\xi}_1 - k_{12}\xi_1 \,,$$

$$\wedge_2 = \sum_{i=1}^{4} C_{yi}\xi_2 - \sum_{i=1}^{4} C_{yi} z\cdot\xi_4 - Q_2\cdot\cos(\Omega t + \psi_2) - k_{21}\dot{\xi}_1 - k_{22}\xi_2 \,,$$

$$\wedge_3 = \sum_{i=1}^{4} C_{zi}\xi_3 - Q_3\cdot\cos(\Omega t + \psi_3) - k_{31}\dot{\xi}_3 - k_{32}\xi_3 \,,$$

$$\wedge_4 = -\sum_{i=1}^{4} C_{yi} z\cdot\xi_2 + \sum_{i=1}^{4}(C_{zi}y^2 + C_{yi}z^2)\xi_4 - M_1\cos(\Omega t + \psi_4) - k_{41}\dot{\xi}_4 - k_{42}\xi_4$$

$$\wedge_5 = \sum_{i=1}^{4} C_{xi} z\cdot\xi_1 + \sum_{i=1}^{4}(C_{xi}z^2 + C_{zi}x^2)\cdot\xi_5 - M_2\cos(\Omega t + \psi_5) - k_{51}\dot{\xi}_5 - k_{52}\xi_5$$

$$\wedge_6 = \sum_{i=1}^{4}(C_{xi}y^2 + C_{yi}x^2)\cdot\xi_6 - M_3\cos(\Omega t + \psi_6) - k_{61}\dot{\xi}_6 - k_{62}\xi_6$$

then the equations of the indignant motions (11) - (13) will look like (7), and stability conditions of system will look like (10).

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